Course Title: DSA 8201 - Foundations for Numerical Techniques

Purpose of the Course: This course aims to provide students with a solid foundation in mathematics, computer science, and programming concepts necessary for learning numerical techniques in data science. By the end of the course, students will have the prerequisite knowledge to pursue advanced courses in numerical methods and computational techniques.

Expected Learning Outcomes: At the end of this course, the student should be able to:

1. Understand and apply fundamental concepts in calculus, linear algebra, and discrete mathematics
2. Demonstrate proficiency in a programming language, such as Python or R, commonly used in data science
3. Implement basic algorithms and data structures relevant to numerical techniques
4. Develop problem-solving skills and critical thinking abilities

Content: Credit Hours: 40 Lecture hours, 20 Lab hours (Credits: 3) Prerequisite: College-level mathematics or equivalent background knowledge

Course Outline:

1. Calculus
   * Functions, limits, and continuity
   * Differentiation and integration
   * Partial derivatives and multiple integrals
   * Series and sequences
2. Linear Algebra
   * Vectors and matrices
   * Matrix operations and properties
   * Eigenvalues and eigenvectors
   * Linear transformations and systems of linear equations
3. Discrete Mathematics
   * Sets, relations, and functions
   * Combinatorics and counting principles
   * Graph theory and network models
   * Recursion and recurrence relations
4. Programming for Numerical Techniques
   * Introduction to Python or R
   * Control structures, loops, and functions
   * Libraries for numerical computation (e.g., NumPy, SciPy)
   * Debugging and error handling
5. Algorithms and Data Structures
   * Algorithm analysis and complexity
   * Basic data structures (arrays, lists, stacks, queues)
   * Sorting and searching algorithms
   * Trees and graphs
6. Problem Solving and Critical Thinking
   * Problem decomposition and abstraction
   * Analytical and logical reasoning
   * Effective communication of solutions
   * Debugging and troubleshooting techniques

Lecture/Dates Topic Intended Learning Outcomes Activities Lecture 1-4 Calculus Students will understand and apply fundamental concepts in calculus - Lecture and problem-solving exercises Lecture 5-8 Linear Algebra Students will learn the concepts and techniques of linear algebra - Lecture and problem-solving exercises Lecture 9-12 Discrete Mathematics Students will explore concepts in discrete mathematics and their applications - Lecture and problem-solving exercises Lecture 13-16 Programming for Numerical Techniques Students will become proficient in Python or R for numerical computation tasks - Lecture and hands-on coding exercises Lecture 17-20 Algorithms and Data Structures Students will implement basic algorithms and data structures related to numerical techniques - Lecture and hands-on coding exercises Lecture 21-24 Problem Solving and Critical Thinking Students will develop problem-solving skills and critical thinking abilities - Lecture, discussion, and group activities

1. Calculus a. Functions, limits, and continuity
   * Functions: A function is a relation between a set of inputs and a set of possible outputs with the property that each input is related to exactly one output. A function is typically represented as f(x).
   * Limits: The limit of a function is the value that the function approaches as the input approaches a certain value. The limit is denoted as lim(x→a) f(x) = L, where L is the limit of the function f(x) as x approaches a.
   * Continuity: A function is continuous at a point x=a if the following three conditions are satisfied: f(a) is defined, lim(x→a) f(x) exists, and lim(x→a) f(x) = f(a). If a function is continuous at every point in its domain, it is said to be a continuous function.

b. Differentiation and integration

* + Differentiation: The derivative of a function f(x) at a point x=a represents the rate of change of the function at that point. The derivative is denoted as f'(x) or df(x)/dx. The derivative of a function f(x) is given by the limit: f'(x) = lim(h→0) [(f(x+h) - f(x))/h].
  + Integration: Integration is the reverse process of differentiation. The integral of a function f(x) represents the area under the curve of the function between two points. The indefinite integral of a function f(x) is denoted as ∫f(x)dx, and the definite integral between points a and b is denoted as ∫[a,b] f(x)dx. The Fundamental Theorem of Calculus connects differentiation and integration: ∫[a,b] f'(x)dx = f(b) - f(a).

c. Partial derivatives and multiple integrals

* + Partial derivatives: In the case of a function with multiple variables, the partial derivative with respect to one variable is the derivative of the function treating all other variables as constants. For a function f(x,y), the partial derivatives are denoted as ∂f/∂x and ∂f/∂y.
  + Multiple integrals: Multiple integrals involve integrating a function with respect to more than one variable. For a function f(x,y), the double integral calculates the volume under the surface defined by the function over a region in the xy-plane. The double integral is denoted as ∬R f(x,y)dxdy, where R represents the region of integration.

d. Series and sequences

* + Sequences: A sequence is an ordered list of numbers, typically generated by a function with a natural number input. A sequence can be denoted as {a\_n}, where a\_n = f(n) and n is a natural number.
  + Series: A series is the sum of the terms in a sequence. A series can be denoted as S = Σ[a\_n], where the sum is taken over a specified range of n values. Common series include geometric series (Σ(ar^n)) and arithmetic series (Σ(a + n\*d)). Convergence of a series is determined by whether the series approaches a finite value as the number of terms increases.